ENGINEERS INSTITUTE OF INDIA

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CHEMICAL ENGINEERING

REVISED AS PER GATE

Instrumentation and Process Control

PROCESS CONTROL

CHEMICAL: GATE & PSUs

ENGINEERS INSTITUTE OF INDIA

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CHEMICAL ENGINEERING

Revised As Per New GATE- Syllabus

STUDY MATERIAL

PROCESS DYNAMICS & CONTROL

Process Dynamics & Control

Marking Analysis in GATE (2000 to 2024)

Year	1 Mark	2 Marks	Total Marks
2024	1×1	2×2	**5**
2023	1×3	2×3	**9**
2022	1×5	2×3	**11**
2021	1×3	2×5	**13**
2020	1×2	2×2	**7**
2019	1×2	2×3	**8**
2018	1×1	2×4	**9**
2017	1×2	2×3	**8**
2016	1×3	2×3	**9**
2015	1×2	2×3	**8**
2014	1×2	2×4	**10**
2013	1×1	2×2	**5**
2012	1×2	2×3	**8**
2011	1×2	2×3	**8**
2010	1×3	2×3	**9**
2009	1×2	2×4	**10**
2008		2×8	**16**
2007	1×1	2×7	**15**
2006	1×1	2×8	**17**
2005	1×4	2×5	**14**
2004	1×1	2×5	**11**
2003	1×3	2×5	**13**
2002	1×2	2×1	**4**
2001	1×3	2×3	**9**
2000	1×4	2×2	**8**

List of Topics in GATE 2024 paper from PDC

(Depreciation)+(Block diagram reduction)(Capitalized cost-K)

CONTENT

Part – V Frequency Response


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3 Rank KRISH PARADVA
4 Rank SMIT MAGANLAL VAGADIYA
5 Rank TUTHIKA MAHESH ANIL
6 Rank PARAM AGRAHARI
6 Rank DEBARGHYA CHAKRABARTI
9 Rank KEVAL VASOYA
11 Rank PRAVEEN KUMAR YADAV
15 Rank BRANZIL
16 Rank NISTHA ROY
39 Rank MANSUR MUSTAFA
42 Rank NITIN SINGH
52 Rank NISHANT PATHAK
and many more ....

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1 Rank
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1 Rank
PRASHANT VARSHNEY

GATE 2016
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RONIKA GOSWAMI

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PART-1:
INTRODUCTION

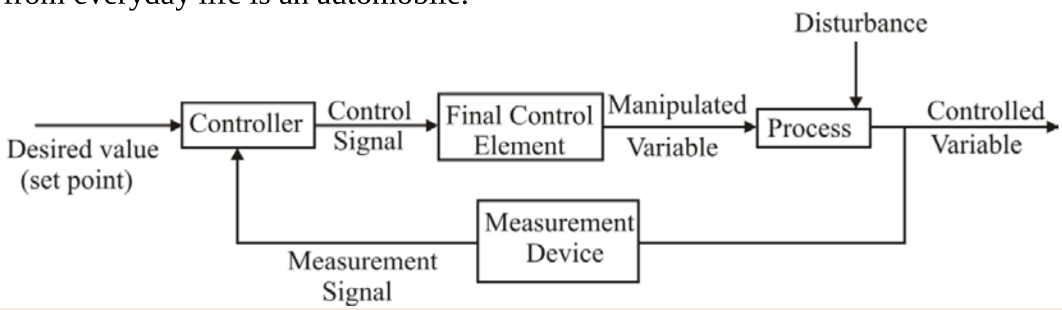
CHAPTER-1:
INTRODUCTORY CONCEPTS

The following topics are covered in the book

- Mathematical tools for understanding the dynamics of process.
- Unsteady response of simple chemical process systems.
- Output of various simple modes of control
- Response of simple systems because of the addition of controllers
- Analysis the stability of controlled systems.
- Introduction to advanced control schemes.

1.2 CONTROL SYSTEMS

Control systems are used to maintain process conditions at their desired values by manipulating certain process variables to adjust the variables of interest. A common example of a control system from everyday life is an automobile.



DEFINITIONS:

Block diagram : Diagram that indicates the flow of information around the control system and the function of each part of the system.

Open loop : In an open loop, the measured value of the controlled variable is not fed back to the controller.

Closed loop : In a closed loop, the measured value of the controlled variable is fed back to the controller.

Controlled variable : The process variable that we want to maintain at a particular value.

Controller : A device that outputs a signal to the process based on the magnitude of the error signal. A proportional controller outputs a signal proportional to the error.

Disturbance rejection : One goal of a control system, which is to enable the system to "reject" the effect of disturbance changes and maintain the controlled variable at the set point.

Disturbances : Any process variables that can cause the controlled variable to change. In general, disturbances the variables that we have no control over.

Error : The difference between the values of the set point and the measured variable.

Manipulated variable : Process variable that is adjusted to bring the controlled variable back to the set point.

Positive feedback : In positive feedback, the measured temperature is added to the set point. (This is usually an undesirable situation and frequently leads to instability).

Negative feedback : In negative feedback, the error is the difference between the set point and the measured variable (this is usually the desired configuration).

Offset : The steady-state value of the error.

Set point : The desired value of the controlled variable.

Set point tracking : One goal of a control system, which is to force the system to follow or "track" requested set point changes.

PART-2:
MODELING FOR PROCESS DYNAMICS

CHAPTER-2:
MATHEMATICAL TOOLS FOR MODELING

Understanding Process Dynamics (how process variables changes with time) will be very important to our studies of Process Control. As we analyse the Chemical Processes, we write material balance and energy balance equations and we find these equations in terms of differential equations. It means linear differential equations arises from mathematical modeling of chemical processes. This will be a common occurrence for us as we continue our studies of process dynamics and control. We can solve these equations by separation and integration. A couple of other useful tools for solving such models are Laplace Transforms and MATLAB / Simulink.

DEFINITION OF LAPLACE TRANSFORM:

The Laplace transform of a function f(t) is defined as F(s) which can be find according to the equation

$$F(s)=\int_0^{\infty} f(t)e^{-st}dt$$

Notation of Laplace transform of $f(t)$ is $\mathcal{L}\{f(t)\}=F(s)$

Example: Laplace transform of function, $f(t)=4$

$$F(s)=\int_0^{\infty} 4e^{-st}dt=\left[\frac{-4e^{-st}}{s}\right]_0^{\infty}=\frac{4}{s}$$

$$\mathcal{L}\{4\}=\frac{4}{s}$$

FACTS ABOUT LAPLACE TRANSFORM:

(1) The Laplace transform is not defined for the function f (t), when the value of '**t**' is less than zero .

(2) The Laplace transform is linear. Mathematically,

$\mathcal{L}\{af_1(t)+bf_2(t)\}=a\mathcal{L}\{f_1(t)\}+b\mathcal{L}\{f_2(t)\}$ Where, a and b are constant.

(3) Laplace transform of the function f(t) exists, if the integral $\int_0^{\infty} f(t)e^{-st}dt$ takes a finite value (i.e. *remains bounded*).

(4) Laplace transform is a transformation of a function from time domain (where time is an independent variable) to s-domain (where, s is an independent variable).
s is a variable defined in complex plane (i.e. s = a + jb)

Use of Laplace Transform :

Laplace transform offers a very simple method of solving linear differential equations. Using Laplace transform, a linear differential equation is reduced to an algebra problem. (Which is simpler than solving differential equation directly).

LAPLACE TRANSFORMS OF SIMPLE FUNCTIONS

(1) The Step Function :

$$f(t)=\begin{Bmatrix} 0 & t<0 \\ A & t>0 \end{Bmatrix}$$

$$\mathcal{L}\{f(t)\}=\int_0^{\infty} Ae^{-st}dt = A\left[\frac{-e^{-st}}{s}\right]_0^{\infty}=\frac{A}{s} \qquad \Rightarrow \mathcal{L}[A]=\frac{A}{s}$$

$$\mathcal{L}[\text{ Step function of size A}] = \frac{A}{s}$$

When function is Unit step i.e. $u(t)=\begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$ $\boxed{\mathcal{L}[u(t)]=\frac{1}{s}}$

(2) The Exponential Function:

$$f(t) = \begin{Bmatrix} 0 & t<0 \\ e^{-at} & t>0 \end{Bmatrix}$$

$$\mathcal{L}\{e^{-at}\}=\int_0^{\infty} e^{-(s+a)t}dt=\left[\frac{-1}{s+a}e^{-(s+a)t}\right]_0^{\infty}$$

$$\mathcal{L}\{e^{-at}\}=\frac{1}{(s+a)}$$

$$\text{Similarly, } \mathcal{L}\left\{e^{at}\right\}=\frac{1}{(s-a)}$$

(3) The Ramp Function:

$$f(t) = \begin{Bmatrix} 0 & t<0 \\ at & t>0 \end{Bmatrix}$$

$$\mathcal{L}\{f(t)\}=\int_0^{\infty} ate^{-st}dt$$

$$\mathcal{L}\left\{f(t)\right\}= a\left[-e^{-st}\left(\frac{t}{s}+\frac{1}{s^2}\right)\right]_0^{\infty}=\frac{a}{s^2}$$

Example: Solve the following equation for x(t),

$$\frac{dx}{dt} = \int_0^t x(t)dt - t$$

$$x(0) = 3$$

Solution: Taking Laplace transform of above equation

$$\mathcal{L}\left[\frac{dx}{dt}\right] = \mathcal{L}\left[\int_0^t x(t)dt\right] - \mathcal{L}[t]$$

$$sX(s) - x(0) = \frac{X(s)}{s} - \frac{1}{s^2}$$

$$sX(s) - 3 = \frac{X(s)}{s} - \frac{1}{s^2}$$

$$\Rightarrow \quad X(s) = \frac{(3s^2 - 1)}{s(s+1)(s-1)}$$

Expanding it by partial fraction method,

$$\Rightarrow \quad \frac{(3s^2-1)}{s(s+1)(s-1)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s-1)} = \frac{A(s^2-1) + B\{s(s-1)\} + C\{s(s+1)\}}{s(s+1)(s-1)}$$

$$\Rightarrow \quad \frac{(3s^2-1)}{s(s+1)(s-1)} = \frac{s^2(A+B+C) + s(C-B) + (-A)}{s(s+1)(s-1)}$$

Comparing the co-efficient on both side,

A + B + C = 3, C – B = 0, -A = -1

We get, A = 1, B =1, C= 1

$$X(s) = \frac{1}{s} + \frac{1}{s+1} + \frac{1}{s-1}$$

By Inverse Laplace Transform, $x(t) = 1 + e^{-t} + e^{t}$

PROPERTIES OF TRANSFORMS:

1. Final value theorem:

If F(s) is the Laplace transform of f(t), then

$$\lim_{t\to\infty} [f(t)] = \lim_{s\to 0} [sF(s)]$$

Provided that sf(s) does not become infinity for any value of s satisfying Re $(s) \geq 0$. the limit of f(t) is found to be correct only if f(t) is bounded as t approaches infinity. The final value theorem allows us to compute the value that a function approaches as $t \to \infty$ when its Laplace transform is known.

Example: Find the final value of the function x(t) for which the Laplace transform is

$$X(s)=\frac{1}{s(s^3+3s^2+6s+8)}$$

Solution : Applying final value theorem,

$$\lim_{t\to\infty}[x(t)]=\lim_{s\to 0}\left[sX(s)\right]=\frac{1}{8}$$

$$\lim_{t\to\infty}[x(t)]=\frac{1}{8}$$

The conditions of the theorem satisfied unless s = –2 or (s+2)≠ 0

2. Initial value theorem:

$$\lim_{t\to 0}\left[f(t)\right]=\lim_{s\to\infty}\left[sF(s)\right]$$

3. Translation of transform: (First shifting property)

If $\mathcal{L}\left[f(t)\right]=F(s)$ then,

$$\mathcal{L}\{e^{-at}f(t)\}=F(s+a)=\int_0^\infty f(t)e^{-(s+a)t}dt$$

4. Translation of function: (Second shifting property)

If $\mathcal{L}\left[f(t)\right]=F(s)$ then,

$$\mathcal{L}\left[f(t-t_0)\right]=e^{-st_0}F(s) \text{ for } t>0$$

Example: Solve the following equation for y (t)

$$\int_0^t y(t)dt=\frac{dy(t)}{dt},\ y(0)=1$$

Solution : Taking Laplace transform,

$$\mathcal{L}\left[\int_0^t y(t)dt\right]=\mathcal{L}\left[\frac{dy(t)}{dt}\right]$$

$$\Rightarrow \quad \frac{1}{s}Y(s)=sY(s)-y(0) \quad \Rightarrow Y(s)=\frac{s}{(s^2-1)}$$

By taking inverse Laplace transform,

$$y(t)=\mathcal{L}^{-1}[Y(s)]=\mathcal{L}^{-1}\left[\frac{s}{s^2-1}\right]=\cosh t$$

Question: A process described by the transfer function $G_p(s) = \frac{(10s+1)}{(5s+1)}$ is forced by a unit step input at time t = 0. The output value immediately after the step input $(\text{at } t = 0^+)$ is __________ (rounded off to the nearest integer). **(GATE-2022, 2-Marks)**

Answer: 2

Example:

Given transfer function

$$G_P(s) = \frac{10s+1}{5s+1} \qquad G(s) = \frac{Y(s)}{X(s)} = \frac{10s+1}{5s+1}$$

$$\text{for step input }, Y(s) = \frac{1}{s}\frac{10s+1}{5s+1}$$

The out value at $t = 0^*$, using initial value theorem

$$\underset{t\to 0}{Y(t)} = \underset{S\to\infty}{SY(s)} = \underset{S\to\infty}{S \times \frac{1}{S}} \times \frac{10s+1}{5s+1}$$

$$\underset{t\to 0}{Y(t)} = \underset{S\to\infty}{\frac{10s+1}{5s+1}} = \underset{S\to\infty}{\frac{10\frac{S}{S}+\frac{1}{S}}{\frac{5s}{S}+\frac{1}{S}}} \qquad \underset{t\to 0}{Y(t)} = \frac{10+\frac{1}{S}}{\underset{S\to\infty}{5}+\frac{1}{S}} = \frac{10}{5} = 2$$

Question: A system has a transfer function $G(s) = \frac{3e^{-4s}}{12s+1}$. When a step change of magnitude M is given to the system input, the final value of the system output is measured to be 120. The value of M is ______________. **(GATE-2021, 2-Marks)**

Ans: 40

$$G(s) = \frac{3e^{-4s}}{(12s+1)}$$

Step change of magnitude M in input, $\overline{X}(s) = \frac{M}{S}$

Final value of the system output = 120

$$\overline{Y}(s) = \frac{M}{S} \times \frac{3e^{-4s}}{(12s+1)}$$

$$\lim_{t\to\infty} Y(t) = \lim_{s\to 0} S.\overline{Y}(s) = \lim_{s\to 0} S \times \frac{M}{S} \times \frac{3e^{-4s}}{(12s+1)} = 120$$

$$\Rightarrow \quad 3M = 120 \qquad M = 40$$

KEY POINTS

(1) Laplace transform of a function f (t), $F(s)=\int_0^{\infty} f(t)e^{-st}dt$, $t > 0$

(2) $\mathcal{L}\{af_1(t)=bf_2(t)\}=a\mathcal{L}\{f_1(t)\}+b\mathcal{L}\{f_2(t)\}$

(3) $\mathcal{L}\left\{\frac{df(t)}{dt}\right\}=sF(s)-f(0)$

(4) $\mathcal{L}\left\{\frac{d^2f(t)}{dt^2}\right\}=s^2F(s)-sf(0)-f'(0)$

(5) For n^{th} order, $\mathcal{L}\left\{\frac{d^nf(t)}{dt^n}\right\}=s^nF(s)-s^{(n-1)}f(0)-s^{(n-2)}f'(0)-..............-sf^{(n-2)}(0)-f^{(n-1)}(0)$

(6) $\mathcal{L}\left\{\int_0^t f(t)dt\right\}=\frac{F(s)}{s}$

(7) Final value theorem, $\lim_{t\to\infty}\{f(t)\} = \lim_{s\to 0}\{sF(s)\}$

(8) Initial value theorem, $\lim_{t\to 0}\{f(t)\}=\lim_{S\to\infty}\{sF(s)\}$

(9) If $\mathcal{L}[f(t)] = F(s)$ then,

$$\mathcal{L}\{e^{(-at)}f(t)\}=F(s+a)=\int_0^{\infty} f(t)e^{-(s+a)t}dt$$

$$\mathcal{L}\{f(t-t_0)\}=e^{-st_0}F(s)$$

Table of Laplace Transforms:

(1.) $\mathcal{L}(1) = 1/s$

(2.) $\mathcal{L}(e^{at})=\frac{1}{(s-a)}$

(3.) $\mathcal{L}(t^n)=\frac{n!}{s^{n+1}}$ when, $t > 0$ & $n \in N$

(4.) $\mathcal{L}(t^n)=\frac{\Gamma(n+1)}{s^{n+1}}$ where, $n \notin N$ n is a fraction.

(5.) $\mathcal{L}(\sin at)=\frac{a}{(s^2+a^2)}$

(6.) $\mathcal{L}(\cos at)=\frac{s}{(s^2+a^2)}$

(7.) $\mathcal{L}(\sinh at)=\frac{a}{(s^2-a^2)}$

(8.) $\mathcal{L}(\cosh at)=\frac{s}{(s^2-a^2)}$

(9.) $\mathcal{L}(e^{at}t^n)=\frac{n!}{(s-a)^{n+1}}$

SALIENT FEATURES OF SINUSOIDAL RESPONSE:

1. The output is a sine wave having some frequency was that of input signal.
2. The magnitude of Amplitude of output signal is less than that of input signal. The output signal is attenuated.

$$\text{Amplitude ratio} = \frac{\text{Output Amplitude}}{\text{Input Amplitude}} = \frac{1}{\sqrt{1+\tau^2\omega^2}} \qquad 0 < AR < 1$$

3. The output lags behind the input by an angle $|\phi|$. The phase lag increases with frequency, but the phase lag can never exceed 90º.

Example-1: A mercury thermometer having a time constant of 0.1 min is placed in a temperature bath at 120ºF and allowed to come to equilibrium with the bath. At time t=0, the temperature of the bath begins to vary sinusoidally about its average temperature of 120ºF with an amplitude of 2ºF. If the frequency is $\frac{10}{\pi}$ cycles/minute. Calculate the temperature reading at 4 minute?

Solution : $\tau = 0.1$ min $x_s = 120^oF$ A = 2ºF

$$f = \frac{10}{\pi}\frac{\text{cycle}}{\text{min}}$$

$$\omega = 2\pi f = 2\pi\frac{10}{\pi} = 20 \text{ rad/min}$$

$$\text{Amplitude of response } = \frac{A}{\sqrt{\tau^2 s^2+1}} = \frac{2}{\sqrt{4+1}} = 0.896$$

$$\text{Phase angle } (\phi) = \tan^{-1}(-2) = -63.5^o$$

Phase lag = 63.5°

Y(t) = 0.896 sin(20t – 63.5°)

y (t) = 120+0.896 sin (20t – 63.5°)

At = 4 minute, y (t) = 120+0.896 sin (20(4) – 63.5°)

y(4) = 120.2544ºF

Example-2: In the temperature alarm unit, a unity gain first order system with a time constant of 5 minutes is subjected to a sudden 50º C rise because of fire. If an increase in 30º C is required to active the alarm, what will be the delay in signaling the temperature change?

Sol. 4.58

$$y_{(t)} = A(1 - e^{\frac{-t}{\tau}})$$

$$30 = 50(1 - e^{\frac{-t}{5}}), \quad 0.6 = 1 - e^{\frac{-t}{5}}$$

$$e^{\frac{t}{5}} = 2.5, \quad \frac{t}{5} = 0.92 \quad \Rightarrow \quad t = 4.58 \text{ minutes}$$

Example-3: The response of a thermocouple can be modeled as a first order process to change in the temperature of the environment. If such a thermocouple at 50ºC is immersed suddenly in a fluid at 120º C and held there, it is found that the thermocouple reading (in º C) reaches 63.2% of the find steady valve in 1.2 minute.
Find the time constant of the thermocouple.

Sol. 71.95

$$y_{(t)} = A(1 - e^{-t/\tau})$$

$$A = 120 - 50 = 70^{o}\ C$$

and $$y_{(t)} = (120 - 50) \times 0.632 = 44.24$$

So, $$\frac{44.24}{70} = 1 - e^{\frac{-72}{\tau}}$$

$$e^{\frac{72}{\tau}} = 2.72 \qquad \frac{72}{\tau} = \ln 2.72 \qquad \tau = 71.95 \text{ sec}$$

Example-4: A process of unknown transfer function is subjected to unit impulse input. The output of the process is measured accurately and is found to be represented by the function y(t) = te^{-t} **.** Determine the unit step response of this process?

Solution:

$$x(t) = \delta(t)$$

$$X(s) = \mathcal{L}\{x(t)\} = 1$$

$$y(t) = te^{-t}$$

$$Y(s) = \mathcal{L}\{y(t)\} = \frac{1}{(s+1)^2}$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)^2}$$

For determining unit step response, $Y(s)=\frac{1}{(s+1)^2}X(s)=\frac{1}{s(s+1)^2}$

Solving by partial function, $\frac{1}{s(s+1)^2}=\frac{A}{s}+\frac{B}{(s+1)}+\frac{C}{(s+1)^2}$

Comparing the coefficient we get A= 1, B = -1, C = -1

$$Y(s)=\frac{1}{s}-\frac{1}{s+1}-\frac{1}{(s+1)^2}$$

Taking inverse Laplace transform, $y(t)=1-e^{-t}-te^{-t}$

KEY POINTS

Key features of standard responses of first-order systems to common inputs.

Input			Output	
	X(t)	X(s)	Y(s)	Y(t)
Step	u(t)	$\frac{1}{s}$	$\frac{k_p}{s(\tau s+1)}$	$K_p\left(1-e^{-t/\tau}\right)$
Impulse	δ(t)	1	$\frac{K_p}{\tau s+1}$	$\frac{K_p}{\tau}e^{-t/\tau}$
Ramp	btu(t)	$\frac{b}{s^2}$	$\frac{bK_p}{s^2(\tau s+1)}$	$K_p\left[bt-b\tau\left(1-e^{-t/\tau}\right)\right]$
Sinusoid	$u(t)A\sin(\omega t)$	$\frac{A\omega}{s^2+\omega^2}$	$\frac{A\omega K_p}{\left(s^2+\omega^2\right)(\tau s+1)}$	$\frac{AK_p\omega t}{1+(\omega\tau)^2}e^{-t/\tau}+\frac{AK_p}{\sqrt{1+(\omega\tau)^2}}\sin\left[\omega t+\tan^{-1}(-\omega\tau)\right]$

CHAPTER-4:
PHYSICAL EXAMPLE OF FIRST ORDER SYSTEM

4.1 EXAMPLES OF FIRST ORDER SYSTEMS:

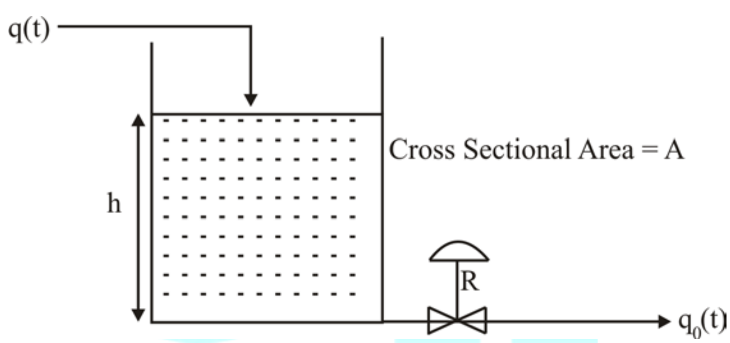
(a) LIQUID LEVEL:

Consider a tank of uniform cross-sectional area A which is attached to a flow Resistance R as valve, a pipe or a weir.

Here q_0, the volumetric flow rate $\left(\dfrac{\text{Volume}}{\text{time}}\right)$ through the resistance, is related to the head h by

$$q_o = \frac{h}{R} \qquad \ldots(4.1)$$

The unit of resistance is time/m^2 . A time varying volumetric flow q of liquid of constant density ρ enters the tank.



We write a mass balance around the tank,

Mass flow in - mass flow out = Rate of mass accumulation of flow in the tank

$$\rho q(t) - \rho q_o(t) = \frac{d(\rho A h)}{dt} = \frac{dm}{dt} \qquad \ldots(4.2)$$

$$q(t) - q_o(t) = A\frac{dh}{dt}$$

Put the value of q_0 (t) form Equation (4.2), we get

$$q - \frac{h}{R} = A\frac{dh}{dt}$$

At Steady state, $\dfrac{dh}{dt} = 0$

Where q_s and h_s are used to convert our system in terms of deviation variables.

$$q_s - \frac{h_s}{R} = 0 \qquad \ldots(4.3)$$

Where the subscript s has been used to indicate the steady state value of the variable.

Subtracting equation (4.3) from equation (4.2) gives

$$(q-q_s)=\frac{1}{R}(h-h_s)+\frac{Ad(h-h_s)}{dt}$$

Assume $Q=q-q_s$, $H=h-h_s$

$$Q=\frac{H}{R}+\frac{AdH}{dt} \qquad \ldots(4.4)$$

Taking Laplace transform,

$$Q(s)=\frac{H(s)}{R}+AsH(s)\Rightarrow Q(s)=H(s)\left[\frac{1}{R}+As\right]$$

It gives, $$\boxed{\frac{H(s)}{Q(s)}=\frac{R}{\tau s+1}} \qquad \ldots(4.5)$$

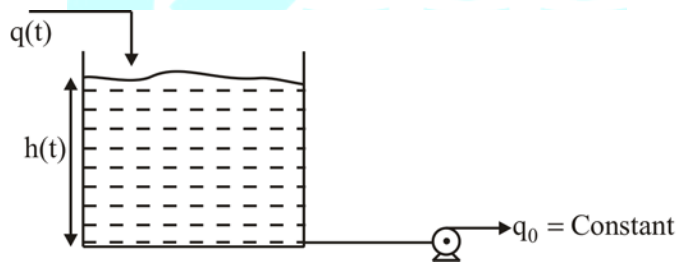
When, $\tau=AR$ and steady state gain , $K_p=R$

(b) LIQUID LEVEL PROCESS WITH CONSTANT FLOW OUTLET:

An example of a transfer function that often arises in control systems may be developed by considering the liquid-level system. The Resistance is replaced by a constant flow pump.

Assumptions:

1. Constant cross sectional area of the tank
2. Constant density of the fluid.



Apply material balance, $$\rho q-\rho q_0=\rho A\frac{dh}{dt}$$

$$\Rightarrow \quad q-q_0=A\frac{dh}{dt} \qquad \ldots(4.6)$$

At steady state,

$$q_s-q_0=0 \qquad \ldots(4.7)$$

Write down the equation (4.6) in deviation variable form,

$$(q-q_s)=A\frac{d(h-h_s)}{dt} \qquad \ldots(4.8)$$

$$\Rightarrow \quad Q=A\frac{dH}{dt}$$

Taking Laplace transform,

$$Q(s)=[sH(s)-H(0)]$$

$$\Rightarrow \qquad Q(s) = AsH(s)$$

$$\boxed{G(s) = \frac{H(s)}{Q(s)} = \frac{1}{As}} \qquad \ldots(4.9)$$

Notice that the transfer function $\frac{1}{As}$ is equivalent to integration. Therefore, the solution is

$$\boxed{h(t) = h_s + \frac{1}{A}\int_0^t Q(t)dt} \qquad \ldots(4.10)$$

Clearly, if we increase the inlet flow to the tank, the level will increase because the output flow remains constant. The excess volumetric flow rate into the tank accumulates, and the level rises. This type of systems is called non-regulation system.

(c) MIXING PROCESS:

Consider a mixing tank of constant hold up volume V in which a stream of solution containing dissolved salt flows at a constant volume flow rate q. The concentration of the salt in the input steam x, varies with time. We find the transfer function relating the outlet concentration y to the inlet concentration x.

We write mass balance around the mixing tank for the salt, flow rate of salt in- Flow rate of salt out = Rate of accumulation of salt in the tank

$$qx - qy = \frac{d(Vy)}{dt} \qquad \ldots(4.11)$$

At steady state, s subscript is used to define steady state variable

$$qx_s - qy_s = 0 \qquad \ldots(4.12)$$

Assume $X = x - x_s, Y = y - y_s$

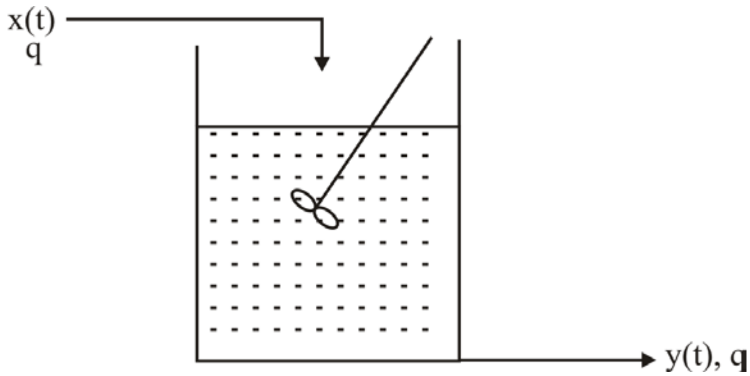


Figure: Mixing tank

Subtracting Equation (4.10) from Equation (4.9) gives

$$qX - qY = \frac{VdY}{dt} \qquad \ldots(4.13)$$

CHAPTER-5:
RESPONSE OF FIRST ORDER SYSTEM IN SERIES

Systems with first order dynamic behavior are not the only ones encountered in a chemical process. An output may change, under the influence of an input, in a drastically different way from that of a first order system, following higher-order dynamics. System with second or higher-order dynamics can arises from several physical situations. These can be classified into three categories:

1. Multi-capacity process: Processes that consist of two or more capacities (first order systems) in series, through which material or energy must flow. Examples:
(a) Non interacting systems
(b) Interacting systems

(a) Non interacting system:

In non interacting system the flow through R_1 depends only on h_1, there is no effect of variation in h_2 in tank 2 on the flow through R_1.

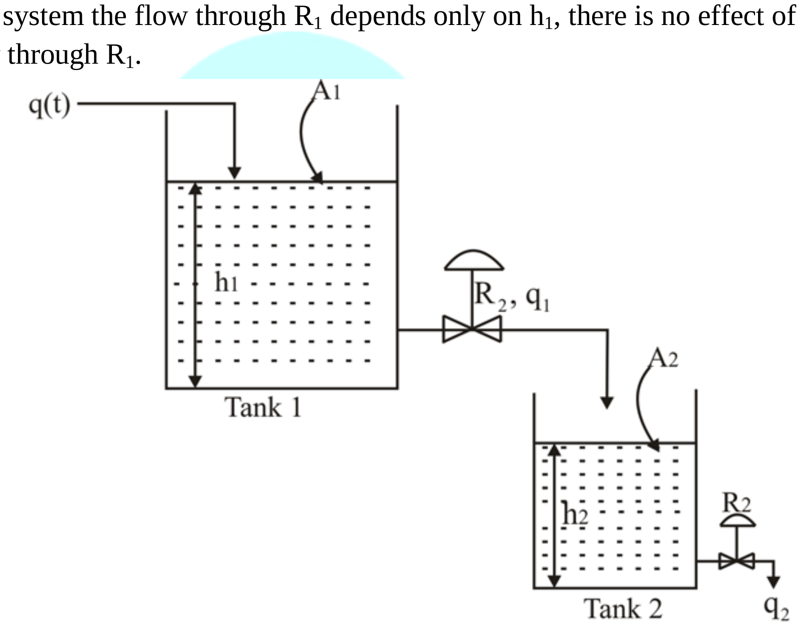


Figure: Two non interacting tank liquid level system

Mass balance on tank1 $$q - q_1 = A_1 \frac{dh_1}{dt}$$ ...(5.1)

At steady state $$q_s - q_{1s} = 0$$ ...(5.2)

Mass balance on tank 2 $$q_1 - q_2 = A_2 \frac{dh_2}{dt}$$ ...(5.3)

At steady state $$q_{1s} - q_{2s} = 0$$ ...(5.4)

CHAPTER-6:
HIGHER ORDER SYSTEM

6.1 SECOND ORDER SYSTEM:

Second order system represents a quadratic lag system. A second order will be developed by considering a classical example from mechanics like damped vibrator .

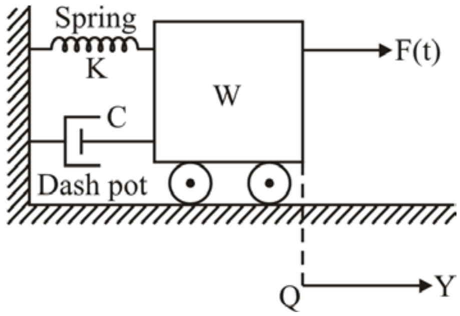


Figure: Damped vibrator

Damped vibrator consist of a block of mass W on a table is attached to a linear spring. A viscous damper is also attached to the block. When force F(t) is applied on the system the system starts oscillating in horizontal direction.

Force acting on block,

1. The force exerted by the spring is Ky. where, K is Hooke's constant
2. The viscous fraction is $C\frac{-dy}{dt}$, where c is damping coefficient.
3. The external force f(t).

Apply Newton's law of motion, $\frac{W}{g_c}\frac{d^2y}{dt^2} = -Ky - C\frac{-dy}{dt} + f(t)$ …(6.1)

Where, W = mass of block , Kg

$g_c = 9.8\frac{m^2}{sec}$

C = viscous damping coefficient, $\frac{Kg}{m.sec}$

K = Hooke's constant

f(t) = driving force

Dividing equation by K gives, $\frac{W}{g_c}\frac{d^2y}{dt^2} + K\frac{C}{K}\frac{dy}{dt} = \frac{f(t)}{K}$

$$\tau^2\frac{d^2y}{dt^2} + 2\zeta\tau\frac{dy}{dt} + y = x(t) \quad ...(6.2)$$

Where, $\tau^2 = \frac{W}{g_c}$, $2\zeta\tau = \frac{C}{K}$

When the system possesses on Inverse Response, its transfer function has at least one positive zero.

Example-01: A system has the transfer function $\frac{Y(s)}{X(s)} = \frac{10}{\left(s^2+1.6s+4\right)}$.

A step change of magnitude units is introduced in this system. What is the percent overshoot and decay ratio?

Answer: Compare, $\left(s^2+1.6s+4\right) = \left(\tau^2 s^2 + 2\zeta\tau s+1\right)$

$$\tau^2 = \frac{1}{4},\ 2\zeta\tau = \frac{1.6}{4} \quad \text{we get, } \tau = 0.5 \quad \zeta = 0.4$$

$$\text{Overshoot} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \exp\left(\frac{-\pi(0.4)}{\sqrt{1-0.42}}\right)$$

Overshoot = 0.254 = 25.4%

Decay ratio = $(\text{overshoot})^2 = (0.254)^2 = 0.0645 = 6.45\%$

Example-02: The transfer function of process is $\frac{1}{\left(8s^2+4s+2\right)}$. If a step change introduced from the system, then what is the response of the system?.

Answer: $G(s) = \left(8s^2+4s+2\right)$

Compare it with, $\left(\tau^2 s^2 + 2\zeta\tau s + 1\right)$

We get, $4 = \tau^2,\ 2\zeta\tau = 2$

$$\tau = 2, \quad \zeta = \frac{1}{2} = 0.5$$

$\zeta < 1$ So, system response is under damped.

Example-03: A system has a transfer function $\frac{2}{\left(3s^2+5s+9\right)}$. If a step change of magnitude 5 is introduced in a system. Determine the ultimate value of y(t) ?

Solution: $\Rightarrow \frac{Y(s)}{X(s)} = \frac{2}{\left(3s^2+5s+9\right)}$

Given, $X(s) = \frac{5}{s}$

$$Y(s) = \frac{10}{s(3s^2+5s+9)}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{10s}{s(3s^2+5s+9)} = \frac{10}{9},$$

then ultimate value of y(t) is $\frac{10}{9}$

GATE Questions

Question-1: A process has a transfer function $G(s)=\frac{Y(s)}{X(s)}=\frac{20}{90000s^2+240s+1}$.

Initially the process is at steady state with $x(t=0)=0.4$ and $y(t=0)=100$. If a step change in $\boldsymbol{x}$ is given from 0.4 to 0.5, the maximum value of y that will be observed before it reaches the new steady state is ___________ (rounded off to 1 decimal place).

Ans: 102.4 to 102.5 **GATE-2021 Marks-2**

$$G(s)=\frac{Y(s)}{X(s)}=\frac{20}{(90000s^2+240s+1)}$$

At Initial $x(t=0)=0.4$

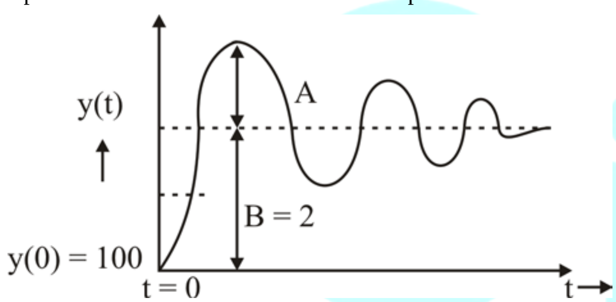
$y(t=0)=100$

Step change in input of magnitude 0.1

$$X(s)=\frac{0.1}{s}$$

$$Y(s)=\frac{0.1}{s}\times\frac{20}{(90000s^2+240s+1)}$$

$k_p=20$ $M=0.1$ $\tau_p=300$ $\varepsilon=0.4$



$B=Mk_p=2$

$$\text{Over shoot}=\frac{A}{B}=e^{\frac{-\pi\varepsilon}{\sqrt{1-\varepsilon^2}}} \qquad A=2\times e^{\frac{-\pi\times0.4}{\sqrt{0.84}}}$$

Maximum value of y = 100 + 2 +A = 102.46

Question-2: A step change of magnitude '4' is introduced into a system having a transfer function :

$$\frac{Y(s)}{X(s)}=\frac{9}{16s^2+3.2s+8}$$

Determine

(a) Percent overshoot

(b) Decay ratio

(c) Period of oscillation

PART-4:
LINEAR CLOSED-LOOP SYSTEMS

CHAPTER-7:
THE CONTROL SYSTEM

7.1 COMPONENTS OF CONTROL SYSTEMS:
To understand a control system we consider an example of a control system for a stirred tank heater as shown in the figure.

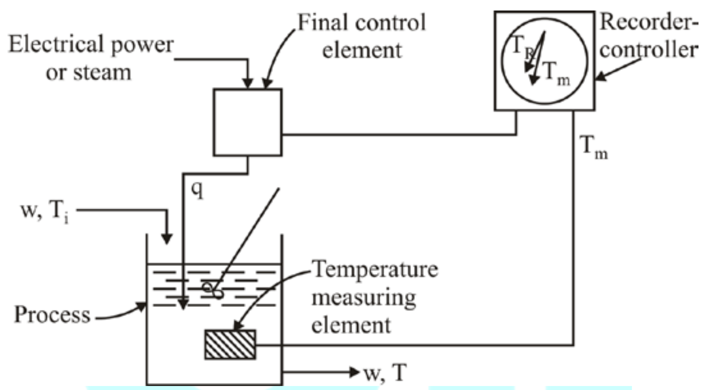


Figure: Control system for a stirred tank heater.

Components of a control system for a stirred tank heater:

1. **Process** (Stirred tank heater): It is a stirred tank heater in which liquid stream having flow rate at a temperature T_i is entering.
2. **Measuring element** (Thermometer): It measures the temperature of the tank (T_m).
3. **Controller:** It senses the difference or error , $\in = T_R - T_m$ .Therefore measures the difference between the measured temperature(T_m) and desired temperature (T_R).
4. **Final control element** (Control valve): It changes the heat input in such a way as to reduce the difference between the desired and measured temperature.

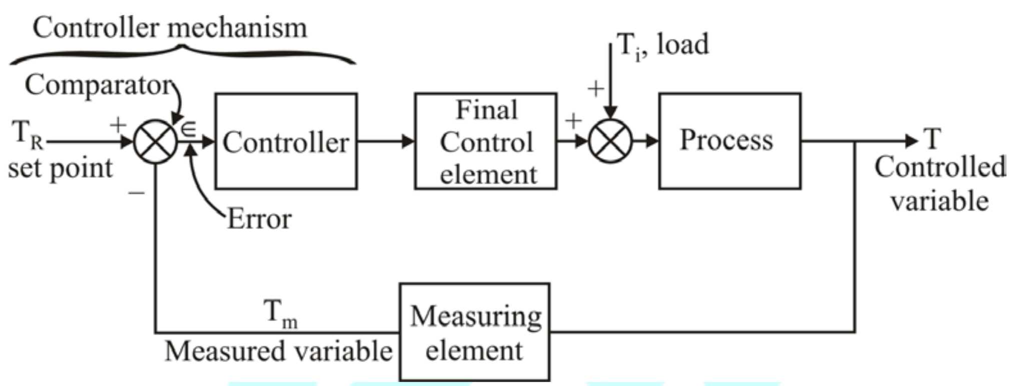
Set point: It is the desired value of the controlled variable.

Load point: It refers to a change in any variable they may cause the controlled variable of the process to change. In the above system, the inlet temperature (T_i) is a load variable.

Block diagram: It is a diagram which makes it much easier to visualize the relationship among the various signal.

7.2 BLOCK DIAGRAM OF A SIMPLE CONTROL SYSTEM

The control system shown in the figure is called a closed loop system because the measured value is return to a device called comparator. Comparator measures the difference between the desired value and measured values and thus an **error** is generated. This **error** enters the controller which adjusts the final control element in order to return the controlled variable to the set point.

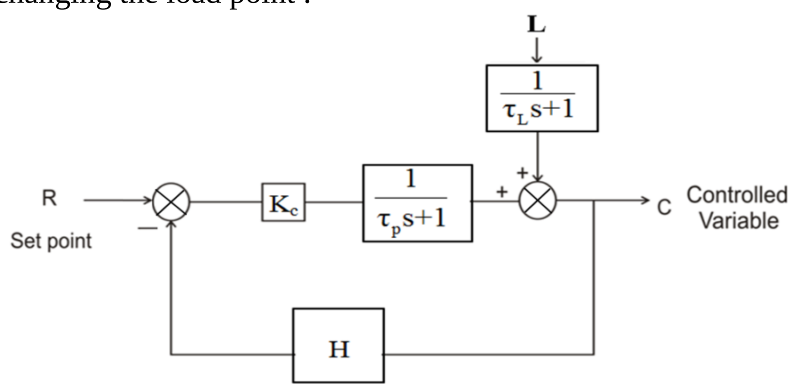


Negative feedback system: It ensures that the difference between T_R and T_m is used to adjust the control element to reduce the error.

Positive feedback system: In this signal to the comparator were obtained by adding T_R and T_m which increases the instability in the system.

Servo Problem: In this type of problem we assume there is no change in load variable T_i and we are interested in changing the set point.

Regulator problem: In this type of problem we assume there is no change in set point and we are interested in changing the load point .



Example: Derive the servo type transfer function for the figure?

Solution: Take Load change = 0,
$$\frac{C}{R}=\frac{K_c\frac{1}{(\tau_P s+1)}}{1+\frac{K_c}{(\tau_P s+1)}H}=\frac{K_c}{(\tau_p s+1+K_c H)}$$

Example: Derive regulator type transfer function for the figure?

Solution: Take set point change = 0,
$$\frac{C}{L}=\frac{\frac{1}{(\tau_L s+1)}}{1+\frac{K_c}{(\tau_P s+1)}H}=\frac{(\tau_P s+1)}{(\tau_L s+1)(\tau_P s+1+K_c H)}$$

KEY-POINTS

1. **Servo Problem:** No Change in load, **change in set point**
2. **Regulator Problem:** No change in set point, **change in load**